*Journal of Engineering Physics and Thermophysics, Vol. 75, No. 5, 2002*

# THERMAL DIAGNOSTICS OF FRICTION IN CYLINDRICAL COUPLINGS. II. COMPUTATIONAL EXPERIMENTS AND GENERALIZATION

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*Results of computational experiments on investigating the efficiency of the algorithm proposed to solve the nonlinear inverse boundary-value problem of restoration of heat release in a plain bearing by measuring the temperature are presented. Simplified three-dimensional models based on the plane thermal model are proposed for practical use of the method of thermal diagnostics of friction in plain bearings as applied to the operated equipment.*

The method of thermal diagnostics of friction in sliding bearings is as follows. A mathematical thermal model quite adequately describing the nonstationary temperature field in a system of contacting bodies is constructed for the movable coupling under study. During the trials, one records the temperature at the internal points of one element of the system at a certain distance from the contact zone. Using the temperature data obtained, by solution of the inverse boundary-value problem one restores the heat release and accordingly the friction power as additional information [1].

The iteration-regularization method employed in solving the inverse boundary-value problem has been substantiated theoretically and studied quite comprehensively as applied to linear ill-posed problems [2]. The algorithm proposed to solve the nonlinear problem of restoration of the moment of frictional forces in a plain bearing has been constructed formally according to the same scheme as for the linear problem. The operating capacity of the algorithm obtained was tested by computational experiments according to the known procedure [2]. The model problem was constructed as follows. The function $Q(t)$ was defined, and the solution $f(\varphi, t)$ of the primal problem at a fixed $r_2 < R < r_3$ in the vicinity of the friction zone was employed as the accurate initial data for solution of the inverse problem. Setting the intensity of heat release in the zone of frictional contact unknown, we restored the function $Q(t)$ with the use of a plane thermal model and the temperature data $T(R, \varphi_j, t) = f(\varphi_j, t)$ and $0 \le \varphi_j \le \varphi_0$, $j = 1, ..., n$.

All the calculations were carried out for a plain bearing (Fig. 1) with the following geometric dimensions: $r_1 = 0.012$, $r_2 = 0.013$, $r_3 = 0.016$, $r_4 = 0.032$, and $\varphi_0 = 12^{\mathrm{o}}$. The bushing in the bearing has been manufactured from filled Teflon for which the temperature dependences of thermophysical properties have the form

$$\lambda_2 = 0.07\,(T - 100)/150 + 0.35 \quad (\mathrm{W/(m \cdot {}^oC)})\,, \quad C_2 = [6 \cdot 10^{-3}\,(T - 30) + 3] \cdot 10^6 \quad (\mathrm{J/(m^3 \cdot {}^oC)})\,.$$

The material for the shaft and the race was steel with the following thermophysical properties:

$$\lambda_1 = 30.5\,(T - 100)/150 + 55.5 \quad (\mathrm{W/(m \cdot {}^oC)})\,, \quad C_1 = [1.2 \cdot 10^{-3}\,(T - 30) + 3.7] \cdot 10^6 \quad (\mathrm{J/(m^3 \cdot {}^oC)})\,.$$

In the case of the inverse problem of restoration of the heat release from the temperature data, the boundary-value problems (primal, conjugate, and for a temperature increase) were solved on each iteration by the finite-difference method. The temperature data were specified for $R = 0.0136$ m and $0 \le \varphi \le \varphi_0$ at the grid nodes. The results of calculating with different numbers of the points of specifying the temperature showed that temperature information at one point along the axis of loading ($\varphi = 0$) suffices to qualitatively restore the heat-release function. Thereafter we carried out all the calculations with measurement of the temperature at one point.

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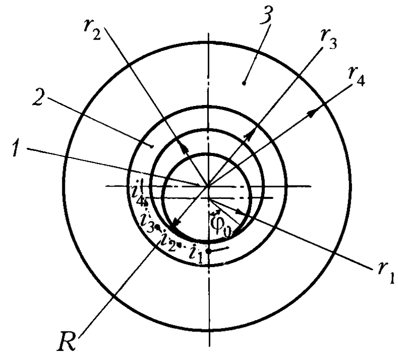


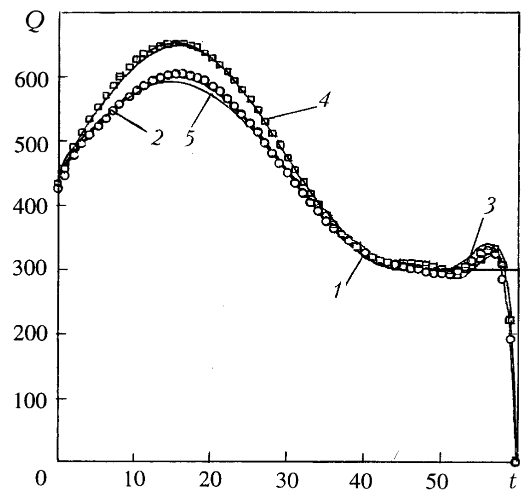


Fig. 1. Scheme of the friction unit: 1) shaft; 2) bushing; 3) race.

Fig. 2. Influence of the averaging of thermophysical properties on the restoration of the function of heat-release intensity: 1) sought $Q(t)$; 2–5) restored $Q(t)$ [2) with allowance for the temperature dependence of the thermophysical properties, 3) for the averaged thermophysical properties, 4) in the case of averaging of only the properties of the bushing material, and 5) in the case of averaging of only the properties of the shaft and the race]. $Q$, W/m; $t$, min.

The results of calculating with accurate initial data demonstrate the stability of the algorithm to errors related to implementation of computational algorithms on a personal computer, which makes it possible to stop the iteration process according to the ordinary condition, for example,

$$\max_t \left| Q^{k+1}(t) - Q^k(t) \right| < \varepsilon . \tag{1}$$

In practical applications, it is of interest to determine in what manner the averaging (used frequently) of thermophysical properties influences the quality of restoration of the heat release and accordingly the moment of frictional force. Figure 2 compares the functions of heat-release intensity which are restored with the use of the averaged (constant) thermophysical properties and with allowance for their dependence on the temperature. The same temperature data corresponding to the nonlinear problem were used.

The calculations showed that in the case of a change of up to 12% in the thermophysical properties of Teflon in the temperature range in question (20–150°C), the averaging leads to a deviation of the calculation results from the exact solution. The averaging of the thermophysical properties of the steel from which the shaft and the race are manufactured influences little the accuracy of restoration of the function $Q(t)$, which is attributed to the smaller relative change (to 2%) in the thermophysical properties of steel. Since the value of the gradient at $t = t_m$ is zero, the sought function $Q(t)$ at the end of the time interval is not refined (it is subtended to the initial approximation), which distorts the quality of restoration in the vicinity of this point. Therefore, the values of the function sought at the end of the time interval which correspond to several time steps can be eliminated from further consideration.

In the actual experiment, the temperature data contain errors, i.e., in addition to the exact part $\bar{f}(0, t)$, the function $f(0, t)$ involves the component of the error $\delta_T = \delta f(0, t)$:

$$f(0, t) = \bar{f}(0, t) + \delta_T . \tag{2}$$

To investigate the influence of different errors in the initial data on solution of the inverse boundary-value problem we simulated the errors using the sensor of random numbers with different distribution laws and superposed them on exact temperature dependences. The solutions of the model problems have shown that, beginning with a certain number, the approximations of the functions of heat-release intensity $Q(t)$ deviate from the solution sought, "ad-

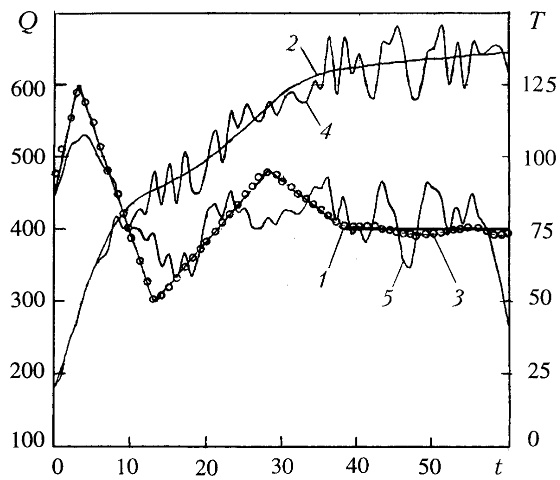


Fig. 3. Restoration of the function of heat-release intensity from disturbed temperature data: 1) sought $Q(t)$; 2) accurate temperature data; 3) $Q(t)$ restored from the accurate temperature data; 4) disturbed temperature data; 5) $Q(t)$ restored from the disturbed temperature data. $Q$, W/m; $T$, $^{\circ}$C; $t$, min.

justing themselves" to disturbed values of the temperature. For large iteration numbers the approximate solution has a strongly oscillating character, which is natural for the iteration solutions of ill-posed problems, among which are inverse boundary-layer problems. In this connection, the process of refinement of the approximate solution was completed according to the condition of iteration regularization when the value of the discrepancy agreed with the quantitative characteristic of the error of the temperature data, i.e., when the condition [2]

$$J\,[Q\,(t)] \le \delta_T^2\,, \quad \delta_T^2 = \int_0^{t_{\mathrm{m}}} \sigma^2\,(t)\,dt\,, \tag{3}$$

where $\sigma^2(t)$ is the variance of the function $f(0,\ t)$, was satisfied. It is assumed that the error of approximation of the boundary-value problem is $\delta_{\mathrm{a}} << \delta_T$ and it can be disregarded. Figure 3 gives the solution of the inverse boundary-value problem which has been obtained with the stopping condition (3) with an error distributed by the normal law with a unit variance and a zero mathematical expectation and constituting 5% of the maximum temperature. We restored the function of heat-release intensity which is characteristic of the time dependence of the moment of frictional force in a plain bearing at constant values of the load and of the sliding velocity (time step is 1 min). The accuracy of restoration of the heat-release intensity is suitable for practical determination of the moment of frictional force in a plain bearing.

The consumption of computer time by solution of the nonlinear multidimensional problem to implement the proposed method of thermal diagnostics of friction is several minutes. In this connection, the method can be employed periodically along with continuous routine methods of diagnostics and monitoring of the technical state to ascertain the correctness of making a decision on the operating capacity of a friction unit. To implement the method of thermal diagnostics in the process of continuous operation of the friction unit it is also necessary to find the distribution of the temperature at the beginning of its measurements. The methods of solution of the inverse retrospective boundary-value problems of simultaneous restoration of the boundary and initial conditions are not sufficiently developed at present. There are only isolated works on solution of such problems in a one-dimensional case (for example, [3]). In this connection, it is necessary to approximately specify the temperature distributions at the instant of time which is considered to be initial and to investigate the possibility of restoring the function of heat-release intensity in the case of approximate specification of the initial condition.

The most natural means implies approximate specification of the initial temperature distribution in the friction unit on the basis of the values of the temperatures measured at one or several points. To test whether such a restoration of the function of heat-release intensity is possible we conducted computational experiments. The model problem

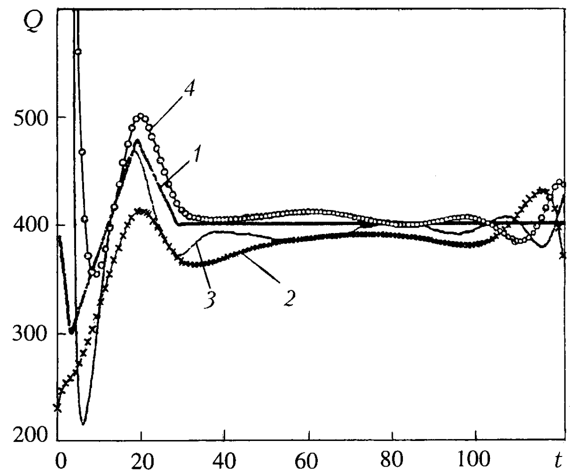


Fig. 4. Restoration of the function of heat-release intensity $Q(t)$ in the case of approximate specification of the initial condition $T(r, \varphi, 0)$: 1) sought $Q(t)$; 2) $Q(t)$ restored at $T(r, \varphi, 0)$ = 83°C; 3) the same, at 50°C; 4) $Q(t)$ restored in linear approximation of $T(r, \varphi, 0)$). $Q$, W/m; $t$, min.

was constructed in two stages as follows. In the first stage, with the known initial condition $T(r, \varphi, 0)$ = 20°C, we defined the intensity function $Q(t)$ and solved the primal problem on a certain time interval (for example, 0–10 min). The temperature distribution in the plain bearing at an instant of time 10 min was stored and employed subsequently as the inhomogeneous initial condition.

In the second stage, we specified the values of the function $Q(t)$ on the time interval [10, $t_m$] and solved the primal problem with the inhomogeneous initial condition obtained in the first stage. The accurate "experimental" data were simulated by the values of the temperature at the point ($R$, 0) which had been obtained from solution of the primal problem. Thereafter we set the function $Q(t)$ unknown and restored it on the time interval [10, $t_m$] from the "experimental" temperature information. Thus, we simulated the switching-on of thermocouples to measure the temperature in a continuously operating friction unit at a certain initial instant of time. For the convenience of interpretation and calculation we shifted the time by 10 min and took again a value of $t = 0$ for the initial time.

The initial condition was specified in different manners. The calculation results (Fig. 4) show a substantial dependence of the quality of restoration on the manner in which the initial condition is specified. In the case of specifying, at all points of the friction unit, a temperature equal to its value at the point of measurement ($R$, 0) at $t = 0$ when we began to record the temperature, the values of the restored function of heat-release intensity (curve 2) turn out to be lower than the sought one virtually throughout the time interval of the trials. This is quite legitimate and explainable. In this case, the specified value of the temperature at the initial instant of time $T(r, \varphi, 0)$ = 83°C is higher than that actual for most of the object of investigation and hence less heat must be expended to obtain the specified temperature at the measurement point. This is confirmed by the specification of an initial condition of 50°C at all the points. Such a value can be obtained by measuring the temperature at a point at a sufficient distance from the contact zone, for example, on the exterior surface of the race along the axis of loading. At the initial instant of time, the specified values of the temperature are also higher than the actual values in the smaller part of the friction unit. The values of the restored function (curve 3) are between the corresponding values of the functions determining the intensity of heat release.

If we measure the temperature at the third point, for example, on the exterior race surface at a point above the maximum gap of the bearing, the initial distribution can be specified by the values of the temperature lying on the plane. In this case the restored value of the heat-release intensity is the closest to the sought one (curve 4).

The above three cases of specifying the initial condition are characterized by a significant deviation of the restored functions from the sought ones in the first 10 minutes. The deviation in the vicinity of the end of the time interval is obtained by virtue of the fact that the functional gradient at $t = t_m$ is zero. Therefore, the values of the functions restored at 4 to 5 points at the beginning and at the end of the time interval can be eliminated. At the remaining points of this interval, the function of heat-release intensity is restored with a reliability sufficient for practical use.

It is obvious that the thermal diagnostics of friction with the use of a plane mathematical thermal model for which the algorithm of solution of the inverse boundary-value problem has been investigated will lead to substantial errors in practical restoration of the entire heat released as a result of friction. In a plain bearing, much of the heat is removed along the length of a metal shaft, which is not taken into account when a plane mathematical thermal model is employed. At the same time, to solve a three-dimensional inverse boundary-value problem one must ensure temperature measurements on a certain surface inside one element, which is impracticable because of the possible discontinuity of the material and distortion of the temperature field. Therefore, in thermal diagnostics of friction, one must construct such mathematical thermal models for a cylindrical coupling whose employment will significantly reduce the number of points of temperature measurement required to restore the moment of frictional force. Furthermore, mathematical thermal models must be constructed under assumptions not limiting practical use.

Let us assume that the temperature distribution is uniform along the bearing length and heat exchange from the bearing and the race is negligibly small. Then in the case where the shaft is rotating with a rather high velocity and the assumption of the uniformity of the temperature field in the cross section of the shaft holds, the temperature field in the plain bearing can be described by superposition of the one-dimensional and two-dimensional equations of heat conduction. The shaft is presented by a one-dimensional rod, while the bushing is presented by a plane element which is orthogonal to the shaft. The equations of different dimensions are related by the condition of heat release in the contact zone. For the plane model in the case in question the condition of frictional heat release in the contact zone yields the heat-conduction equation for the distribution of the temperature in the shaft with a heat source in the region of contact

$$C_1(U)\frac{\partial U}{\partial t}=\frac{\partial}{\partial z}\left(\lambda_1(U)\frac{\partial U}{\partial z}\right)-\frac{2\,[\pi-\theta(z)\,\varphi_0]\,r_1}{S_1}\,\alpha_1(U-T_{\mathrm{amb}})+$$

$$+\left[Q(t)+2r_2d\int_0^{\varphi_0}\lambda_2(T)\left.\frac{\partial T(r,\varphi,t)}{\partial r}\right|_{r=r_2}d\varphi\right]\frac{\theta(z)}{S_1d}\,; \tag{4}$$

$$T(r_2,\varphi,t)=U(z_{\mathrm{cont}},t)\,,\quad z_{\mathrm{cont}}\in A\,,\quad |\varphi|\le\varphi_0\,, \tag{5}$$

where $\theta(z) = 1$ for $z \in A$ and $\theta(z) = 0$ for $z \not\ni A$.

The temperature distribution in the bushing and the race satisfy the two-dimensional nonstationary equation of heat conduction in cylindrical coordinates with traditional boundary conditions of the third kind on free surfaces. With such modeling of the thermal process in a plain bearing, to solve the inverse boundary-value problem on determination of the intensity of heat release and the corresponding moment of frictional force it suffices to have temperature measurements at one point, since this problem is analogous to the plane one.

In the case where the heat flux around the circle of the shaft can be taken to be uniformly distributed but the temperature distribution is nonuniform in the cross section, the conditions in the friction zone are written in the form

$$2\varphi_0r_1\int_{z_1}^{z_2}\lambda_1(U)\left.\frac{\partial U(r,z,t)}{\partial r}\right|_{r=r_1}dz\;-\;2r_1d\int_0^{\varphi_0}\lambda_2(T)\left.\frac{\partial T(r,\varphi,t)}{\partial r}\right|_{r=r_2}d\varphi=Q(t)\,; \tag{6}$$

$$T(r_2,\varphi,t)=U(r_1,z,t)\,. \tag{7}$$

Conditions (6) and (7) relate the two-dimensional nonstationary equation of heat conduction over $r$ and $\varphi$ for the bushing and the race to the equation over $r$ and $z$ of the shaft. To solve the inverse boundary-value problem it also suffices to measure the temperature at one point of the bushing.

In the case of reverse motion of the shaft with a small amplitude the temperature in the contact zone will be nonuniform over the angular coordinate; the temperature field in the shaft is described by the three-dimensional heat-conduction equation, while the temperature field in the bushing with a race is described by the two-dimensional equation. The conditions of heat release in the friction zone can be written in the form

$$\lambda_1 (U) \left. \frac{\partial U (r, \varphi, z, t)}{\partial r} \right|_{r=r_1} - \lambda_2 (T) \left. \frac{\partial T (r, \varphi, t)}{\partial r} \right|_{r=r_2} = \tilde{Q} (\varphi, t) . \quad (8)$$

To restore the function $\tilde{Q}(\varphi, t)$ we must measure the temperature near the contact zone around the circle within the angle of contact.

Thus, the plane mathematical thermal model provides the basis for construction of simplified three-dimensional thermal models which enable one to restore the friction moment in a plain bearing from the temperature measurements at a practical number of points. The universality of the method of iteration regularization employed to solve the inverse boundary-value problem enables one to easily generalize all basic relations for different versions of a mathematical thermal model.

This work was carried out with financial support from the Russian Foundation of Basic Research "r98arktika", project No. 00-01-96205.

## NOTATION

$Q$, intensity of heat release in the zone of frictional contact, W; $\tilde{Q}$, specific intensity of heat release, W/m$^2$; $f$, known temperature information, $^{\circ}$C; $j$ and $n$, No. and number of points of temperature measurement; $J$, discrepancy functional; $T$, bearing temperature; $T_{\text{amb}}$, ambient temperature, $^{\circ}$C; $U$, shaft temperature, $^{\circ}$C; $t$, running time, sec; $t_{\text{m}}$, trial time, sec; $r$, $\varphi$, $z$, cylindrical coordinates; $\varphi_0$, half-angle of contact of the bushing with the shaft; $i_{k_2}$ ($k$ = 1, 2, ...), points of temperature measurement for a fixed radius $R$; $S_1$, cross-sectional area of the shaft, m$^2$; $C_i$ ($i$ = 1, 2), heat capacity of the material of the shaft (race) and the bushing, respectively, per unit volume, J/(m$^3\cdot^{\circ}$C); $\lambda_i$ ($i$ = 1, 2), thermal conductivity of the material of the shaft (race) and the bushing respectively, W/(m$\cdot^{\circ}$C); $\alpha_1$, coefficient of heat transfer from the shaft surface, W/(m$^2\cdot^{\circ}$C); $d = z_2 - z_1$, bearing length, m; $A$, set of the points $z_{\text{cont}}$ of contact of the shaft with the bushing. Subscripts: amb, ambient; cont, contact.

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